# Transformation Of Momentum energy and force.

Let there are two frames of reference S and S'. S' is moving with respect to S with translational velocity v.

Let u and u' be the velocities of the particle in S and S' frame.

Suppose in S frame the component of momentum are $p_x = mu_x = \frac{m_0 u_x}{\sqrt{1-u^2/c^2}}$, $p_y = mu_y = \frac{m_0 u_y}{\sqrt{1-u^2/c^2}}$

$$p_z = \frac{m_0 u_z}{\sqrt{1-u^2/c^2}} \longrightarrow (1)$$

Again $p_x' = mu_x' = \frac{m_0 u_x'}{\sqrt{1-\frac{u'^2}{c^2}}}$ ——

Now $u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$

and $\sqrt{1-\frac{u'^2}{c^2}} = \frac{\left[\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)\right]^{1/2}}{1-\frac{u_x v}{c^2}}$

Using the values in the expression of $p_x'$

$$p_x' = \frac{m_0 u_x'}{\sqrt{1-\frac{u'^2}{c^2}}} = \frac{m_0\left(1-\frac{u_x v}{c^2}\right)}{\sqrt{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}} \cdot \frac{u_x - v}{\left(1-\frac{u_x v}{c^2}\right)}$$

$$\boxed{p_x' = \frac{m_0(u_x - v)}{\sqrt{(1-u^2/c^2)(1-v^2/c^2)}}} = \frac{m_0 u_x - v m_0}{\sqrt{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}}$$

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$$P_x' = \frac{m_0 u_x - m_0 v}{\sqrt{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)}} = \frac{m u_x - m c^2 v/c^2}{\sqrt{1-v^2/c^2}}$$

where $m = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}$

$$\Rightarrow P_x' = \frac{P_x - vE/c^2}{\sqrt{1-v^2/c^2}} \longrightarrow (2)$$

$$P_y' = \frac{m_0 u_y'}{\sqrt{1-u'^2/c^2}} = \frac{m_0 \frac{u_y\sqrt{1-v^2/c^2}}{(1-u_x v/c^2)}}{\sqrt{\left(1-u^2/c^2\right)\left(1-v^2/c^2\right)}} \times \left(1-\frac{u_x v}{c^2}\right)$$

$$\Rightarrow P_y' = \left(\frac{m_0 u_y}{\sqrt{1-\frac{u^2}{c^2}}}\right) \times \left(1-\frac{u_x v}{c^2}\right)$$

$$\Rightarrow P_y' = m u_y = P_y \qquad \Rightarrow P_y' = P_y \longrightarrow (3)$$

$$\text{and } P_z' = P_z \longrightarrow (4)$$

## The transformation for energy :

Let $E'$ be the energy of the particle of mass $m'$ in the $S'$ frame and $E$ be the energy of the particle of mass $m$ as measured from the frame of reference $S$.

$$E' = m'c^2 = \frac{m_0 c^2}{\sqrt{1-u^2/c^2}} = \frac{m_0 c^2 / \left(1-\frac{u_x v}{c^2}\right)}{\sqrt{\left(1-u^2/c^2\right)\left(1-\frac{v^2}{c^2}\right)}}$$

$$\Rightarrow E' = \frac{m}{\sqrt{1-v^2/c^2}} \left(1-\frac{u_x v}{c^2}\right)$$

$$E' = m'c^2 = \frac{m_0 c^2}{\sqrt{1 - u'^2/c^2}} = \frac{m_0 c^2 \left(1 - \frac{u_x v}{c^2}\right)}{\sqrt{\left(1 - v^2/c^2\right)\left(1 - \frac{u^2}{c^2}\right)}}$$

$$= \frac{mc^2 - m u_x v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \frac{E - v p_x}{\sqrt{1 - v^2/c^2}}$$

Now the energy-momentum tranformation relation can be written as.

$$E' = \frac{E - v p_x}{\sqrt{1 - v^2/c^2}} \longrightarrow \frac{E'}{c} = \left(\frac{E}{c} - \frac{v}{c} p_x\right) \Big/ \sqrt{1 - \frac{v^2}{c^2}}$$

$$p_x' = \frac{p_x - v E/c^2}{\sqrt{1 - v^2/c^2}}$$

$$p_y' = p_y$$

$$p_z' = p_z$$

Force :- Let a particle of mass m be under the action of force F and have a momentum $\vec{p}$ in an inertial frames. Then

$$F = \frac{dp}{dt} \quad \therefore \quad F_x = \frac{dp_x}{dt}, \quad F_y = \frac{dp_y}{dt} \quad \text{and} \quad F_z = \frac{dp_z}{dt} \longrightarrow (1)$$

$F_x$'s and $p_x$ are the component of force and momentum respectively.

The corresponding values in s' frame moving relative to s frame with velocity v along x (or x') axis are

$$F_x' = \frac{dp_x'}{dt'}, \quad F_y' = \frac{dp_y'}{dt'}, \quad F_z' = \frac{dp_z'}{dt'} \longrightarrow (2)$$

We have the tranformation relation for momentum and time are as follows.

$$p_x' = \gamma\left(p_x - \frac{vE}{c^2}\right)$$
$$p_y' = p_y$$
$$p_z' = p_z$$
and $t' = \gamma\left(t - vx/c^2\right)$

$$\therefore F_x' = \frac{dp_x'}{dt'} = \frac{\gamma\left(dp_x - \frac{v\,dE}{c^2}\right)}{\gamma\left(dt - \frac{v\,dx}{c^2}\right)}$$

$$= \frac{dp_x/dt - \frac{v}{c^2}\,dE/dt}{1 - \frac{v\,dx/dt}{c^2}}$$

$$F_x' = \frac{F_x - \frac{v}{c^2}\,dE/dt}{1 - \frac{v u_x}{c^2}} \longrightarrow (3)$$

Now $E^2 = p^2c^2 + m_0^2c^4$

$\Rightarrow 2E\frac{dE}{dt} = 2p\frac{dp}{dt}c^2$

$\Rightarrow \frac{dE}{dt} = u \cdot F\,c^2$ where $p = mu$

$\Rightarrow \frac{dE}{dt} = u \cdot F \longrightarrow (4)$ $\qquad \frac{dp}{dt} = F$

using (4) in (3)

$$F_x' = \frac{F_x - \frac{v}{c^2}(u \cdot F)}{1 - \frac{u_x v}{c^2}}$$

$$= \frac{F_x - \frac{v}{c^2}(u_x F_x + u_y F_y + u_z F_z)}{1 + \frac{u_x u}{c^2}}$$

$$= \frac{F_x\left(1 - \frac{u_x v}{c^2}\right) + \frac{v}{c^2}(u_y F_y + u_z F_z)}{\left(1 - \frac{u_x v}{c^2}\right)}$$

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$$= \frac{F_x - \frac{v}{c^2}(u_y F_y + u_z F_z)}{\left(1 - \frac{u_x v}{c^2}\right)}$$

Agang $F_y' = \frac{dp_y'}{dt'} = \frac{dp_y \sqrt{1 - v^2/c^2}}{dt\,(1 - u_x v/c^2)} = \frac{F_y \sqrt{1 - v^2/c^2}}{1 - u_x v/c^2}$

Ug $F_z' = \frac{F_z \sqrt{1 - v^2/c^2}}{1 - u_x v/c^2}$

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